Chemical reaction-diffusion implementation of finding the shortest paths in a labyrinth

N. G. Rambidi* and D. Yakovenchuk
Physics Department, Moscow State University, Moscow, Russia
(Received 20 July 2000; published 23 January 2001)

An experimental technique for finding the shortest paths in a labyrinth is elaborated on based on chemical reaction-diffusion media. The system designed has hybrid architecture that combines an information-processing reaction-diffusion medium performing operations of high computational complexity with a digital computer carrying out supplementary operations. Two principal points are assumed as a basis for this design. They are the following: a light-sensitive Belousov-Zhabotinsky-type reagent chosen as a reaction-diffusion medium that offers the opportunity to simulate a labyrinth and spread wave evolution by its images stored in the medium; fast light-induced phase wave processes that spread through the labyrinth in seconds instead of the dozens of minutes typical of trigger waves inherent in reaction-diffusion media. Images of consecutive wave-spreading steps are stored in the memory of a digital computer. These images are used to determine the shortest paths based on the additional procedure of testing for the connectedness of labyrinth fragments.

DOI: 10.1103/PhysRevE.63.026607 PACS number(s): 03.50.−z, 07.05.Bx, 07.05.Pj, 05.45.−a

I. INTRODUCTION

Nonlinear reaction-diffusion media comprise some of the more promising embodiments of parallel and distributed computing. Early theoretical considerations on the information-processing capabilities of these media (see, for instance, [1–4]) were launched in the late eighties. Almost simultaneously, numerical calculations [5–8] showed that reaction-diffusion systems seemed to be an effective tool for solving problems of high computational complexity. The decisive step towards experimental implementation of the information-processing capabilities inherent in these media was made by Kuhnert *et al.* [9–11], who proposed using the light-sensitive reaction-diffusion media of Belousov-Zhabotinsky type. These media proved to be effective for the investigation of image-processing operations (see [12] and references therein).

Given its information-processing aspects, the nonlinear chemical reaction-diffusion medium could be regarded as a highly parallel multiprocessor system. Pseudoflat versions of these systems (thin flat layers of Belousov-Zhabotinsky reagent) are similar in their architecture and capabilities to distributed Grossberg-type neural networks having lateral connections [12,13]. It is desirable, therefore, to look for methods for effectively solving other problems of high computational complexity based on reaction-diffusion media. The most promising is the problem of finding the shortest paths in a labyrinth.

II. SEVERAL REMARKS ON THE PROBLEM OF FINDING THE SHORTEST PATHS IN A LABYRINTH

The problem of searching for a path in a labyrinth determined by certain conditions specifically initiated by biological information processing is one of the most well known contemporary problems of high computational complexity. There has been a variety of attempts to find effective algorithms for solving this problem (see, for instance, [14] and references therein). A labyrinth could be considered an object whose topological properties are associated with a finitely oriented graph. This means that the object should be composed of arbitrary number of vertices and edges connecting them. Let us divide the vertices into four types: starting points that are entrances into the labyrinth (the power of the vertex is equal to 1), intermediate points (vertex power $\geqslant 2$), deadlocks and target points (the vertex power equal to 1).

The simplest in their structure graphs are trees [Fig. 1(a)]. They have one starting point and arbitrary numbers of branches and target points. Multigraphs containing cyclic combinations of edges are more complicated [Fig. 1(b)]. In this case at least two routes might be found between chosen starting and target points. Proposals to use reaction-diffusion media for solving labyrinth problems have been extensively discussed over the last decade and a half (see, for instance, [14–20]). Extremely high parallelism of reaction-diffusion dynamics was the gist of these proposals. The most important of them was experimental work by Steinbock, Toth, and Showalter [17], who investigated navigating a complex labyrinth based on the trigger-wave processes of the Belousov-Zhabotinsky media. Regrettably, conclusions on the practical applicability of these reaction-diffusion systems were pessimistic [18] because of the very low velocity of trigger waves in these media.

This paper is an attempt to show that (a) known light-sensitive reaction-diffusion media of Belousov-Zhabotinsky type can be effectively used nowadays to solve at least simple labyrinth problems, and (b) the effective technique could be applied to finding the shortest paths in a labyrinth on the basis of information on consecutive steps of wave spreading through the labyrinth.

The basic features of the approach used were discussed earlier [21] based on primitive labyrinth structure. Its application for finding paths in complicated labyrinth structures based on improved techniques for data interpretation will be considered in detail below.

III. FUNDAMENTALS OF THE APPROACH

Three principal points were assumed in our technique for finding the shortest paths in a labyrinth.

*Author to whom correspondence should be addressed. Electronic address: rambidi@polly.phys.msu.su

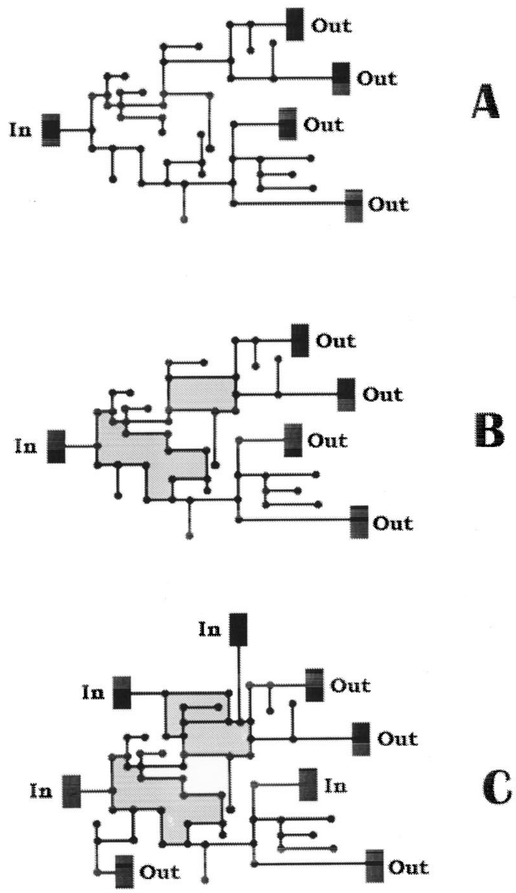

FIG. 1. Labyrinth structures of different complexity. *A*—simple tree-type labyrinth; *B*—tree-type labyrinth containing cycles (filled with gray color), *C*—complicated labyrinth containing cycles and arbitrary numbers of starting and target points.

(i) The information-processing system based on reaction-diffusion mechanisms and capable of solving labyrinth problems should be of hybrid type. It should be a combination of reaction-diffusion medium with a universal digital computer. This architecture makes it possible to effectively perform operations of high computational complexity (parallel spreading of a wave along all pathways of the labyrinth, etc.), together with fast digital processing of intermediate data. Let us make some remarks important in understanding how to design an effective computational procedure for finding paths in a labyrinth. Given the computational character of the procedure, the labyrinth should be stored in memory as an image (in the simplest case, as a black and white image, for instance, a black representation of the labyrinth on a white background). The starting point of a labyrinth should be defined, and a procedure designed, such as to allow the opportunity to record consecutive steps of the wave spreading to the computer memory. When the wave spreads along the path of the labyrinth, the black color of the labyrinth path changes to the color of the background. More-or-less sophisticated procedures could be developed that enable one to follow the wave front and to determine the point where the black labyrinth path would disappear. These points should be deadlocks or target points. However, there is no possibility of distinguishing between them. Therefore, a starting point and target points should be defined so as to find paths in a labyrinth. The starting point and target points should be known. Suppose that a trigger wave spreads along the labyrinth beginning from its starting point. Eventually the wave would reach the target point nearest to the starting point. After that, other target points would be successively determined. It is easy to see that only relative *lengths* of the paths could be determined, not the paths themselves. This shows the principal feature of the computational procedure for finding paths in a labyrinth based on wave processes inherent in nonlinear reaction-diffusion media. The spreading of the wave through a labyrinth is a parallel operation of high computational complexity. A reaction-diffusion medium is capable of performing this operation effectively, and consecutive steps could be stored in the memory of a digital computing system. It will be shown below that the computational digital procedure for low computational complexity can be extended to find paths in a labyrinth, based on the data stored in the computer memory that describe wave spreading through the labyrinth.

(ii) The most important problem involves storing a labyrinth of arbitrary structure in the reaction-diffusion medium for further processing. Steinbock, Toth, and Showalter [17] solved this problem by cutting out rectangular regions of the membrane where a ferroin catalyst was immobilized. This procedure allowed the realization of a variety of geometries and provided an effective two-dimensional system. Steinbock, Kettunen, and Showalter [22,23] prepared the reaction-diffusion system by imprinting the image of an object on the surface of a membrane, using a solution of Belousov-Zhabotinsky catalyst instead of ink. However, these methods are not good enough for a computing device that should be capable of rapidly reorganizing labyrinth images in the course of its processing and of changing labyrinths of different structures (see below). The light-sensitive Belousov-Zhabotinsky-type reagent seems to be one of the most suitable media for solving labyrinth problems because the key feature of light-sensitive excitable media is that they store input information for a rather long period of time [12]. The initial labyrinth image and steps in its further transformation in the medium can be detected by a video camera and stored in the memory of a digital computer.

(iii) The main and decisive issue of the problem under consideration is how to organize a wave process capable of spreading in a parallel mode through all possible paths of a labyrinth. Two kinds of traveling processes inherent in reaction-diffusion systems are known [24,25]. The first of these represents propagation of trigger waves due to the interaction of chemical reaction and diffusion of reaction components. The velocity of the trigger waves is very low ($\sim$0.05 mm/s). The second of these, the so-called phase waves, propagate independently of diffusion along a phase gradient in an oscillatory medium. The phase waves are fast, but regretably they do not transport information. The important point is that control of Ru-based Belousov-Zhabotinsky dynamics through light radiation could be used to organize a wave process moving along a labyrinth.

The actual picture projected on the surface of the medium is a combination of the chosen black and white image (initial labyrinth image) and arbitrary halftone diffuse light background produced by the optical system. If the background is uniform, the input image emerges simultaneously at all points in the medium and changes simultaneously at all points in the process of its evolution from one state to another. Otherwise, if the background is not uniform an additional uncontrolled light component adds to the input image and the process of the image emergence becomes more complicated. The dynamics of the Belousov-Zhabotinsky reaction is controlled by a concentration of Br^- ions produced during the photochemical process initiated by light radiation. Therefore, the moment of image emergence at some point in the medium and the beginning of image evolution at this point is determined by the total exposure produced by the initial image and uncontrolled nonuniform background. This effect is seen in the experiment as additional waves spreading along the gradient of the uncontrolled background.

Let us design a nonuniform background that decreases monotonically from some starting point to some target point. If this background is projected on the layer of Ru-based Belousov-Zhabotinsky reagent, the information on characteristics of the background is introduced into the medium. It is stored as a distribution of the reaction components (Br^- concentrations). In the process of image evolution in the medium, the darkest points of the background emerge first, followed by all other points where concentrations of the Br^- ions are less dense. This process generates a light-induced phase wave. This wave does not transport information because the information was previously introduced into the medium at all points between the starting and target points. This wave makes information visible, transforming it from spatial into temporal form.

Let the self-oscillating mode of the Ru-based Belousov-Zhabotinsky reaction be chosen suitable for solving the labyrinth problem when negative and positive images of the input image alternate in a process of temporal image evolution. Suppose also that the quality of the optical system is high enough and the amount of uncontrolled background negligibly small. In this case, both the negative and positive images of the input image emerge simultaneously at all their points during the process of image evolution.

Let a controlled nonuniform background of some predetermined shape be superimposed on an initial labyrinth image (see Fig. 2) and that this combined image is used instead of the initial image. Then a light-induced phase wave controlled by the shape of the background will spread along the paths of the labyrinth. The shape of the background should be chosen so as to make the time of the wave spreading less than a lifetime of the labyrinth negative image in the medium. This process could be recorded by a video system and its successive realizations could serve as initial data for finding paths in the labyrinth.

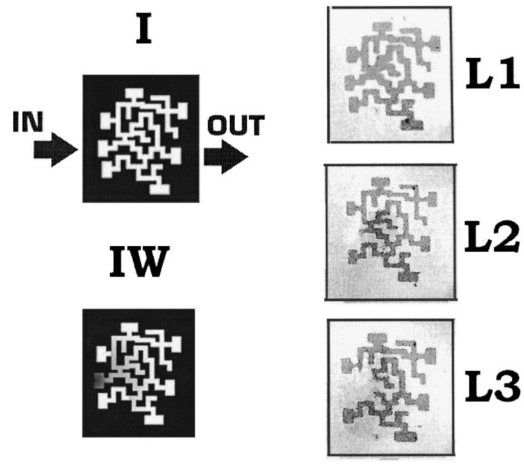

FIG. 2. Initial labyrinth image (I), a nonuniform background superimposed on the initial labyrinth image (IW), and the first stages of labyrinth evolution in the Belousov-Zhabitinsky-type reaction-diffusion medium (L1–L3).

IV. EXPERIMENT TECHNIQUE

The experiment setup [26] is shown in Fig. 3. The principal feature of this setup is the computer controlled Sanyo PLC-510M LCD video projector (VGA-compatible, 270 ANSI lumens). A high level of uniformity of the background intensity of this projector improved the reliability of the experiment. At the same time, the computer-controlled projector was indispensable in the extension of the technique to finding the shortest paths in complex labyrinths (see below).

The reaction-diffusion medium was a thin (0.5–1.5-mm) flat, nonstirred reagent layer placed in a reaction vessel in which a spatiotemporal oscillating process was proceeding. The initial concentrations of the reagent components were $KBrO_3$ (0.3 M),H_2SO_4 (0.5 M), malonic acid (0.2 M), KBr (0.05 M).

Light-sensitive catalyst $Ru(bpy)_3Cl_2$ was immobilized on the surface of the solid support placed at the bottom of the reaction vessel. SILUFOL UV254 plates for thin layer liquid chromatography were used. They were placed into

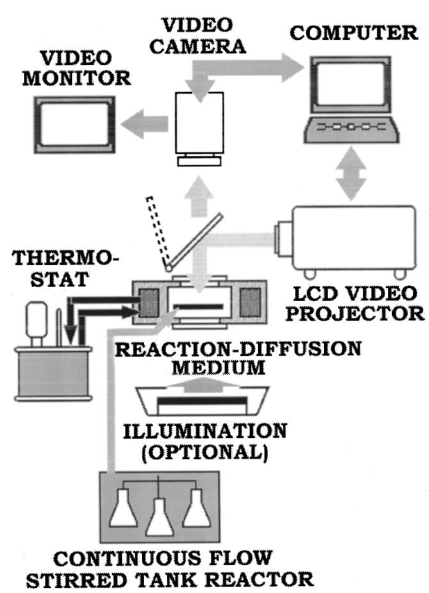

FIG. 3. Principal setup scheme for finding the shortest path in a labyrinth.

~0.0001-M solution of the catalyst for a half an hour to immobilize the catalyst. An oscillatory dynamic mode with negative and positive image alternation proved to be the optimum one for solving labyrinth problems.

An excitation of a thin planar layer of the medium by light radiation was used for the input of initial information. The direction of radiation flow was normal to the surface of the medium. The distribution of light intensity on the surface (that is, an image of the labyrinth under processing) determined the initial image stored in the medium.

It was easy to extract information from the medium by illuminating it with white light over 15–30 s.

V. PROCEDURE FOR FINDING THE SHORTEST PATHS IN A LABYRINTH

The procedure for finding the shortest paths in a labyrinth consisted of two basic stages. The first of these was exciting the phase wave at a chosen point of the labyrinth and recording consecutive steps of the wave spreading through the labyrinth. The images of the labyrinth corresponding to these steps were stored in the computer memory. The second stage was numerical processing of these images to determine the shortest path between starting and target points of the labyrinth.

Let us begin our discussion of the technique starting with the case of the simplest linear-tree-like labyrinth having one starting point and several target points. The initial labyrinth picture is shown in Fig. 4.

Let a predetermined nonuniform and monotonically decreasing background be superimposed at a chosen point of the initial labyrinth image. After projecting this combined image onto the surface of the Belousov-Zhabotinsky reagent, its negative image appears in the medium. At the inception, an image of the initial labyrinth emerges (black image on white background). After that, a spreading wave appears, changing black color to background color along the labyrinth pathway.

The spreading time of this wave through the labyrinth depends on the slope of the superimposed background intensity. It is easy to make it rather small (about 3–5 s) and shorter than the lifetime of the negative image state in the process of its evolution in the medium from negative to positive state. The wave spreading in the labyrinth beginning at the starting point of the labyrinth to its target points is shown in Fig. 4. Since this process is taking place over the course of about 3–5 s, it is easy to record consecutive steps of this spreading with a video camera and store these records in the computer's memory. Some of these records are shown in Fig. 4.

The search for the shortest path from the starting point to the target point was performed using numerical image processing of stored records. Proposals on how to use wave propagation through a labyrinth to find the shortest paths are known [18–20] (see also references in [20]).

In the process of wave spreading when the wave passes over a branching point the labyrinth is divided into two (or more) fragments (see Fig. 5). One of these is connected to the output, but the other one is not. It is easy to find the fragment connected to a target point of the labyrinth if a backward wave is initiated in the medium at this point. As a result, fragments connected to a target point change their color (from black to the color of labyrinth background) while the color of the nonconnected fragment remains unchanged.

If the complexity of the labyrinth is not too high, it is possible to change this auxiliary reaction-diffusion process

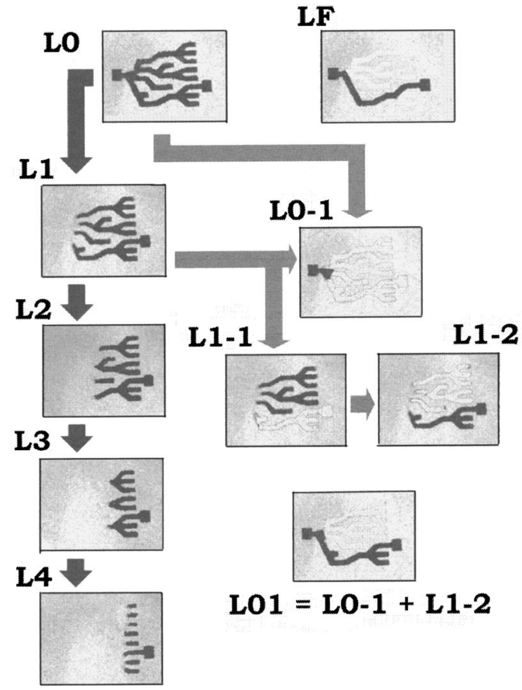

FIG. 4. Finding the shortest path in a simple tree-type labyrinth. L0 is an initial image of the labyrinth in the Belousov-Zhabotinsky-type medium, L1–L4 are some consecutive steps in its evolution during the process of wave spreading, LF is the image of the shortest path in the labyrinth, L0-1 is the pathway that the wave has taken during the first step of its spreading, L1-1 is the result of paint-bucket operations for the L1 image, L1-2 presents the result following subtraction of nonconnected with target point parts from the L1 image (see details in text).

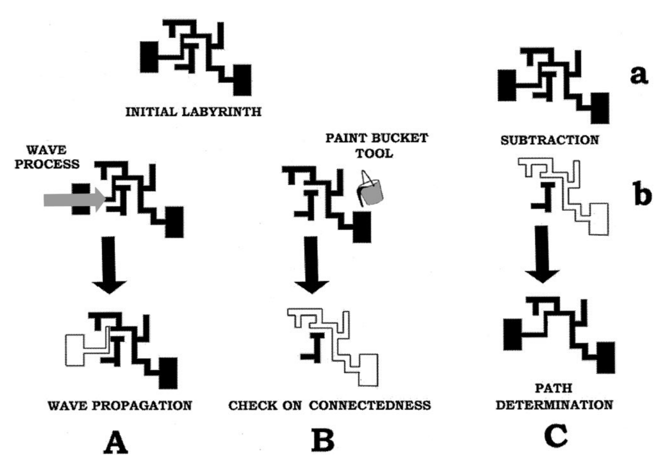

FIG. 5. Basic steps in the procedures for finding the shortest paths in a labyrinth. *A*—wave propagation; *B*—check for connectedness; *C*—removal of deadlocks (subtraction of *B* from *A*).

by numerical processing of the images of the labyrinth stored in the memory of the computer, specifically, to use the filling of black fragments by the background color (paint-bucket operation) initiated at the target point of the labyrinth. Subtracting the image obtained after paint-bucket operation from the initial labyrinth image makes it possible to remove the fragment not connected with the output (Fig. 5)

All image-processing operations used for the implementation of the procedure discussed can be effectively performed using Adobe Photoshop 5.0 software.

Successive repetition of this procedure at every branching point allows the opportunity to exclude all blind channels (and paths to other possible target points) and to determine the path from the starting to the target point.

Let us make two important remarks concerning the procedure suitable for finding the shortest paths in linear-tree-like labyrinths. First, the advantage of this procedure is that there is no need to determine the position of the branching point with a human being as an operator. The shortest path can be found in this case as a result of reiterated steps that consecutively process records of the wave spreading.

The sequence of operations for each step is the following (suppose that processing of the records begins from the first, L1; see Fig. 4).

(a) Subtracting record L1 from the initial image of the labyrinth L0 to determine the pathway that the wave has taken from inception to the considered points of wave spreading (L0-1);

(b) filling fragments of L1 connected with a target point with background color (L1-1);

(c) subtracting L1-1 from L1, that is, rejecting parts not connected to a target point (L1-2; note that subtraction of some fragment from the background intensity should correspond to the zero level of intensity);

(d) adding L0-1 and L1-2 to determine the running image of the labyrinth (L0-1) to be used at the next step instead of L0;

(e) changing L0 to L0-1.

The results of the operations corresponding to every step which is the running image of the labyrinth at the step considered would be (1) the same as the running image of the labyrinth at the previous step if the wave did not pass over a branching point; (2) a changed running image, where some parts of the labyrinth are rejected if the wave has passed over a branching point.

The second important remark is that changing the backward wave through paint-bucket operation to determine the parts of the labyrinth not connected to a target point is not correct because paint-bucket operation is not parallel. However, it is possible to use it if the complexity of the labyrinth is not very high and the time of labyrinth processing is not too long.

The procedure discussed is simple and effective in the case of linear-tree-like labyrinths where all possible pathways from the starting point to the target points have nearly the same directions as the direction of the wave spreading.

In general, however, this procedure should be modified because light-induced phase waves spread along the gradient of the background intensity, not along the pathway of the labyrinth, which can significantly change its direction.

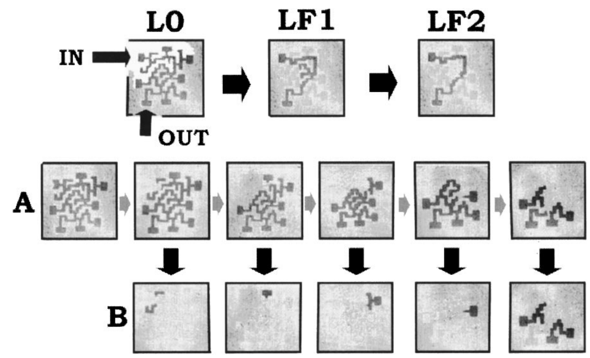

FIG. 6. Finding the shortest path in an arbitrary labyrinth. L0 is the initial image of the labyrinth in the Belousov-Zhabotinsky-type medium, *A* shows the stages of its evolution, LF1 and LF2 are images of the shortest paths (see details in text), *B* shows the results of paint-bucket operations.

Changes in the procedure offered allow the opportunity to use its basic principles and to evolve a technique suitable for even complicated labyrinths.

This step-by-step technique is based on dividing the labyrinth into linear-tree-like parts and on reiterative processing of each of them. Each individual step of the technique consists of the following operations.

(1) determining the labyrinth pathway direction at a chosen starting point;

(2) exciting a phase wave at a chosen starting point that would spread along the gradient of the background intensity, which coincides in this case with the labyrinth pathway direction, and recording consecutive steps of the spreading path;

(3) rejecting possible blind pathways;

(4) determining the pathway turning points.

Each step ends at the pathway turning point, which is further considered as the starting point for the next step.

The most important operation of this step-by-step technique is the determination of the pathway turning points. The results shown below were obtained by using visual control of the process of finding paths in a labyrinth. At the same time, algorithms can be offered for the implementation of this technique. A computerized version that involves carrying out the step-by-step procedure without a human being as an operator is in progress now and will be published elsewhere. Let us mention as well that the main idea of moving step by step along a labyrinth pathway is analogous to a numerical-simulation approach successfully developed by Munuzuri and Chua [21] to find the shortest paths in an arbitrary complex labyrinth. An example of finding the shortest path in a labyrinth based on the technique discussed is shown in Fig. 6.

The most important feature of this case is that the labyrinth under investigation is of multigraph type, containing a cyclic combination of edges. Therefore, the path between starting and target points contains, after using the step-by-step technique, some part of this cycle (see Fig. 6, LF1). However, this part of the cycle disappears (see Fig. 6, LF2) if the step-by-step technique is used repeatedly, taking the shortest path obtained (Fig. 6, LF1) as the new labyrinth.

VI. SOME CONCLUDING REMARKS

There are several important problems in the practical implementation of our technique and in estimating its efficiency. The main operational feature of the Ru-based Belousov-Zhabotinsky system is that it is highly sensitive to small variations in experimental conditions. The quality of the optical system used for the input of initial data should therefore be high. Moreover, the reaction vessel should be thoroughly protected from outside light radiation.

The most important condition determining the quality of the images produced by the Belousov-Zhabotinsky medium is the invariance of the thickness of the medium layer at all points. Two methods of medium formation satisfying this condition were described in [26].

The first of these was the use of a uniform layer of polyacrylamide gel saturated with Belousov-Zhabotinsky reagent. The second, used in this study, was the formation of a pseudo-two-level system. The catalyst in this system is immobilized on the surface of a solid support (a thin layer of silica gel). All other reaction components are in the liquid phase and the reaction itself proceeds at the boundary of the phases.

Let us note in addition that the Ru-based Belousov-Zhabotinsky reaction has a serious drawback, which is that optical density variations in the course of reaction are too small in comparison with ferroin-catalyzed reactions. As a consequence, the reliability of the technique decreases because the quality of the images recorded by a video camera is inferior. To avoid this drawback, a small concentration of ferroin catalyst was immobilized additionally on the surface of the support after immobilization of the Ru catalyst. This made it possible to sufficiently increase the optical density of the images and to use photochemical input of the initial data.

Let us make some remarks on the efficiency of the technique discussed above.

When a wave goes over a branching point, some part of the labyrinth is removed as a result of the procedure used; the more complicated the labyrinth, the bigger the parts that are rejected in the process of finding the shortest paths. Therefore, the remarkable feature of the technique offered is that its efficiency level becomes higher if the complexity of the labyrinth increases. The step-by-step procedure significantly increases the time necessary to find the shortest paths in a labyrinth. The efficiency of the procedure depends on the operational time of the reaction-diffusion medium and on the number of labyrinth turning points in this case. Nevertheless, it is sufficiently higher than the efficiency of procedures using trigger waves [17,18]. The time necessary for processing a labyrinth of average complexity is about 5 min, assuming the cycle time of the medium to be about 40 s. This time is shorter by one order of magnitude in comparison with the time of the trigger-wave procedure [17].

Another important feature of this procedure is that the operational time is linear with respect to the number of the labyrinth pathway turning points.

ACKNOWLEDGMENT

The authors would like to express their sincere gratitude to Professor A. R. Khokhlov for his interest and valuable contributions to this paper.

[1] N. G. Rambidi, V. M. Zamalin, and Yu. M. Sandler, J. Mol. Electron. **4**, S39 (1988).
[2] Ch. Zulicke, W. Ebeling, and L. Schimansky-Geier, BioSystems **22**, 261 (1989).
[3] V. I. Krinsky, V. N. Biktashev, and I. R. Efimov, Physica D **49**, 247 (1991).
[4] A. V. Holden, J. V. Tucker, and B. C. Thompson, Physica D **49**, 240 (1991).
[5] C. B. Price, P. Wambacq, and A. Oosterlinck, IEE Proc.-I: Commun. Speech Vision **137**, 136 (1990).
[6] S. Ei and M. Mimura, SIAM (Soc. Ind. Appl. Math.) J. Math. Anal. **21**, 346 (1990).
[7] K. Kunish and H. Schelch, J. Math. Biol. **27**, 633 (1989).
[8] G. Turk, Computer Graphics **25**, 289 (1991).
[9] L. Kuhnert, Nature (London) **319**, 393 (1986).
[10] L. Kuhnert, Naturwissenschaften **73**, 96 (1986).
[11] L. Kuhnert, K. I. Agladze, and V. I. Krinsky, Nature (London) **337**, 244 (1989).
[12] N. G. Rambidi and A. V. Maximychev, BioSystems **41**, 195 (1997).
[13] S. Grossberg, Neural Networks **1**, 17 (1988).
[14] A. I. Adamatzky, Math. Comput. Modelling **23**, 105 (1996).
[15] M. Conrad, R. R. Campfner, K. G. Kirby, E. N. Rizki, G. Schleis, R. Smalz, and R. Trenary, BioSystems **23**, 175 (1989).
[16] A. Babloyantz and J. A. Sepulchre, Physica D **49**, 52 (1991).
[17] O. Steinbock, A. Toth, and K. Showalter, Science **267**, 868 (1995).
[18] K. Agladze, N. Magome, R. Aliev, T. Yamaguchi, and K. Yoshikawa, Physica D **106**, 247 (1997).
[19] V. Perez-Munuzuri, V. Perez-Villar, L. O. Chua, IEEE Trans. Circuits Syst. **40**, 174 (1993).
[20] A. P. Munuzuri and L. O. Chua, Int. J. Bifurcation Chaos Appl. Sci. Eng. **8**, 2493 (1998).
[21] N. G. Rambidi and D. V. Yakovenchuk, BioSystems **51**, 67 (1999).
[22] O. Steinbock, P. Kettunen, K. Showalter, J. Phys. Chem. **100**, 18 970 (1996).
[23] O. Steinbock, P. Kettunen, and K. Showalter, Science **269**, 1857 (1995).
[24] E. J. Reusser and R. J. Field, J. Am. Chem. Soc. **101**, 1063 (1979).
[25] K. Agladze, S. Obata, and K. Yoshikawa, Physica D **84**, 238 (1995).
[26] N. G. Rambidi, T. O.-O. Kuular, and E. E. Machaeva, Adv. Mater. Opt. Electron. **8**, 163 (1998).